PRESENTED AT ENGINEERING SERVICES
GENERAL ELECTRIC
COMPUTER SEMINAR
SEPTEMBER 7-8, 1955
WALDORF-ASTORIA HOTEL—NEW YORK CITY

G. A. PHILBRICK

G. A. Philbrick Researches, Inc.

Modern Analog Computing Machines

Their Precepts and Purposes

INTRODUCTION

Naturally I appreciate this opportunity of reviewing the available analog computing instruments for an audience of this character. It is especially an honor since my company is one of about a dozen competitive manufacturers of this type of equipment. As a result, I am bound by your trust to avoid the role of a protagonist. By a firm application of will power and ethical principle, I have now been officially transmuted into a completely impartial and unprejudiced commentator. Thus, for example, the fact that Philbrick Researches makes the best computing instruments on the market today does not mean that I am going to favor its products by one little bit over any others. Furthermore I hope that my greater familiarity with our own devices and doctrines will not in itself be taken as evidence that I am biased.

My assignment here, as I take it, is not to cover the theory of analog computers, or the philosophy or scientific morphology of the various general kinds of computing devices. Other presentations on this program have dealt with these aspects of the subject more ably than I could hope to do. My job is assumed to be that of describing the nature and application of existing analog machines, presumably of the electronic type. While I will shortly undertake this, I think it will be helpful to make a brief excursion for the purpose of bringing these machines a little closer to home. Certainly we have little to fear from analog computers. They are friendly natural phenomena - like ourselves though not nearly as complicated - and probably they have a good deal more to fear from us than we from them.

It goes without saying that all of you are technologically very advanced. My exposition nevertheless is going to be pretty much down-to-earth. It will be expressed in familiar forms, and will make use of familiar concepts, so that you should find no difficulty in passing on to your associates - at whatsoever technical levels - those portions of this material, if any, which you consider valuable. To these ends I should like to introduce two principal viewpoints, or attitudes, with regard to analog computing. The first may be designated COMPUTERS AS MODELS: the second will be COMPUTERS AS INSTRUMENTS.

THE ANALOG COMPUTING MACHINE AS A MODEL

If you build any sort of model, of a structure or of a phenomenon, then in the broadest sense you have conjured up an analog computer. Conversely, every analog computer is truly a model, both by definition and in every other practical sense.

Consider for a moment what are called "scale models," such as those used to study a building, earthwork, or vehicle. These models generally perform in the same physical media as do the original objects, and are pretty directly analogous to the latter in ways which are more or less obvious. Perhaps most often the scale model is substantially smaller than its prototype, the handier size then being one of the principal reasons for its existence. On the other hand such a model may be vastly larger than the original, as in the case of modelling crystal lattices or molecules themselves. In either case, the space dimensions undergo a transformation, usually in some rather definite ratio, in passing from the original to the model. Similarly, factors like time, mass, force, and temperature may be transformed, still without changing from the general physical character of the original. The scale model may run faster or slower, hotter or colder, whichever serves the purposes at hand. This technique constitutes a well-established and tremendously useful tool of engineering, which our fancier bag of tricks will never displace, whether in pilot plants, in the towing tank, or in the wind tunnel.

We cannot exhaustively catalog all the many types of models which engineers and other technical men have worked up out of necessity and active imaginations. Please note, however, that in all cases they involve a transformation, or scaling, of their dimensions and variables as referred to some full-scale entity: original or contemplated. Furthermore, these transformations all serve the purpose of making the model more convenient, in terms of accessibility for manipulation, for measurement of behavior, and for changing the conditions and parameters which go to make up the functional character of the system or structure being studied.

We may now introduce the Differential Analyzer - progenitor of all modern analog machines - as a slightly more remote scale model, in which the transformations involved in passing to the model are no longer simply scale changes, but may include transformation of the physical character of the variables of the system. Thus most of the variables may be converted into, and represented by, mechanical rotations. Time may keep its normal form, and even its normal pace, if the Analyzer is a Simulator (cf. infra), or may undergo a change of pace. It may also be re-embodied as a mechanical rotation. Again, time in the model may represent some entirely different variable in the original. But my point is that the normal intuitive concept of a model may thus be extended to include the Differential Analyzer, and thence by a not unreasonable stretch of the imagination to all analog machinery however resplendent or mysterious. I can honestly recommend this viewpoint, as a guide to the applicability and the administration of this kind of automatic computation.

The type of analog computing machine in widest use today is often called the Electronic Differential Analyzer. We can almost say that this machine is simply a differential analyzer in which electrical voltage (instantaneous DC) has replaced mechanical rotation, but this is a careless oversimplification. A more detailed account should bring out the fact that the true differential analyzer was a considerably more general device, both mathematically and dynamically. It is truly hard to believe that this mechanism is a thing of the past. Meanwhile considerations of size and speed have translated gears and shafts into a paradise of electronics.

One class of analog machinery, almost entirely in the category of electronic computers, needs no special effort as regards proving the propriety of its identification with models. This class includes what are known as Simulators, which are chiefly distinct from self-respecting computing machines only by virtue of their frankly avowed imitative function. In this case the model argument is superfluous; the fact is that it is pretty generally hard to tell a simulator from a more orthodox computer unless you notice what is connected to it. The distinction is simply that the computer by connotation is a self-contained model of an entire - or arbitrarily isolated - system, whereas a simulator is a model only of part of a system, the remainder of the system being made up of "real" devices, such as human beings, experimental or production-run control mechanisms, and so on. The simulator, if it is representative and reliable, may substitute for an aircraft, a reactor, or a prime mover, to which actual instrumentation may be connected by appropriate transducers. Experimentation may now go forward with little inhibition, since a badly-conceived design or a lapse of skill in adjustment will only turn on a limit indicator or ring a bell. The worst catastrophe is a tempest in a teapot.

It should be pointed out in all fairness that while our main line of discussion here is directed toward general purpose model devices, this approach should by no means be allowed to overshadow the many important special-purpose techniques and equipment, for example in the department of physical models. Witness the electrolytic tank for the study of field problems, and the conducting sheet methods for similar purposes, not to mention some AC and DC network analyzers and a great many other approaches which are aimed at particular jobs, in which as often as not they can beat the more generally conceived machines all hollow. It will suffice for present purposes to point out the kinship, in the kingdom of models, of these special analogs, in which the variables may or may not appear in their original forms, to the more general object of our present attentions.

This might also be an ideal opening in which to air an opinion regarding the use of passive networks in combination with the operational equipment to be described in what follows. The conjunction of these powerful tools makes possible feats which may be awkward for either one alone, and this is not sufficiently recognized by the engineering public at large. But back to our argument.

THE ANALOG COMPUTING MACHINE AS AN INSTRUMENT

It impresses me that the idea of a computer as simply an instrument is a useful piece of rationalisation. Of course you may find my emphasis of this viewpoint somewhat trivial, especially if you already look on computers as instruments, but there seems to be a tendency just now to make something more awesome of these machines, or of their newer embodiments, and the aura of a higher type of organism strikes me as leading one away from the proper utilitarian view of a computing device as a tool. I may personally represent a minority on this question, since about half of my working life has been spent in the development of what are generally called "instruments", rather than on computing apparatus. My outlook on engineering is more or less that of an instrument man, who naturally considers such an outlook both wholesome and desirable. Thus it is normal for me to think, and I am ready to defend this attitude, that instruments in the broad sense as tools of technology, stand as the most powerful means for progress in any endeavor. More particularly, my own experience with computing devices leads me to consider them not only as instruments, but as the most potent agency available for the development of instruments themselves.

The instrument engineer, whether he is manufacturer or user, enjoys classifying all instruments into indicating, recording, and controlling types. This division leads to a rationalization which is more fundamental than may appear on the surface.

Indication (or measurement), Recording, and Controlling; the importance and universality of these three basic instrumental operations may be dramatized by pointing out that the Present may be measured, the Past may be recorded, and the Future - within reason - may be controlled. You will furthermore agree that the mechanisms and lore of these three functions in the world of instruments are familiar and well understood. May I now state that nothing more than these operations is involved in the most complex and recondite of computing machinery.

Parenthetically, these remarks on the identification of computing devices as instruments apply as much to digital as to analog types. The necessary operation of storage, for example, amounts certainly to recording. Counting itself is as basic a measurement as may be exhibited. But I am afield!

The instrumental technique of control, and I intend this term to mean automatic control, lies at the heart of almost every accurate operation. We cannot here digress into the phenomenon of Feedback, or to the slightly more general subject of causally-closed loops, except to say that it is basic in every automatic control and regulatory system, and in every null-seeking instrument. It is also the key to precision in analog computing, contributing thereto by the substantial removal, at each stage, of the influence of all parts of the local system except for the characteristics of those accurate circuit components which are chosen to determine quantitative operating characteristics.

Among the principal analog computing operations are adding, integrating, and multiplying. Instrumentally, then, a computing machine will possess subordinate units which will measure

sums, integrals, and products of input signals which are supplied to them. These measured quantities are the outputs of such units. In operation, the units are connected in such a way that their individual inputs are either supplied from the outside, or provided directly from the outputs of other units within the machine. The outputs of these units, similarly, serve to furnish other internal inputs, to give outputs of the whole machine, or both. Thus the analog computer is just a somewhat ramified instrument.

Naturally the arrangement of the interconnections will depend on the problem to be solved. In the simplest cases a single unit may comprise the whole computer, as when the job is to integrate or differentiate a varying DC voltage. In a rather complex case there may be no inputs at all from the world outside the instrument. In such a case the machine, in addition to being an instrument, may also be a self-contained model in which a phenomenon is being reproduced in complete isolation, except for occasional manipulation and observation by the operator.

THE OPERATIONAL AMPLIFIER: AN INSTRUMENT WITHIN

The three fundamental computing operations already referred to, among many others, are generally carried out to high precision by means of the so-called Operational Amplifier. Each unitary computing structure in which it is included constitutes an automatic control system, with the amplifier serving as the primary controlling relay. (Fig. 1) Almost universally, the operational amplifier is DC, like the major input and output signals of the computing units. It operates in conjunction with a network of resistors and capacitors (plus occasionally other passive elements such as diodes), and works to maintain a DC voltage null at a special point in the network. The amplifier output is usually also the output of the unit operation, thus giving a "low impedance", or independence of whatever loads may be encountered in operation. The external operational input is normally made direct to the network.

BASIC COMPUTING UNIT

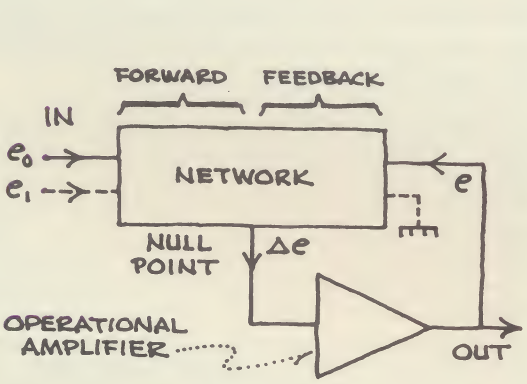


Figure 1

This feedback technique is hardly the exclusive property of analog computing, but abounds in servos, in governors, and many other regulatory artifices. Here, as elsewhere, certain rules must be recognized if the desired criteria of prompt operation plus stability are to be satisfied together.

Operational Amplifiers are available in a variety of forms, ranging from very simple to very complex. They may have a single input, or a pair of balanced differential inputs, though they normally have just one output. One may select from a variety of output power capabilities. Stabilities referred to the input may be within 1V, 0.1V, or 0.01V, and contact stabilisation may be included to extend the input stability down below 0.001V. Naturally the size and cost will depend to some extent on these specifications.

Depending on the nature of the above network, this operational unit may be a linear amplifier, an adder, a time-integrator, etcetera. Note that the amplifier draws negligible circuit from the DC junction, and has (negative) voltage gain from 10^2 up to 10^8.

MATHEMATICAL OPERATIONS AND A PRACTICAL CASE

The operational technique just introduced, involving an amplifier and a cooperating network, leads to an impressive hierarchy of mathematical operations. In computing lore, the local analog structures for these operations are frequently shown explicitly, with the amplifier and circuit elements intact. Sometimes, however, the local circuits are symbolized in block diagram form, in which the blocks represent the various elementary mathematics in a more general way (Fig. 2). The two languages do not conflict, and a direct translation is feasible - as we can show in some fundamental cases. In departing from the amplifier symbolism, as in block diagrams which use the more general operations, we are careful to show the causal order by means of arrows on the signal paths. The A, C, J convention, while it has caught on rather widely, is still somewhat special. I am therefore obliged to show a quite general set of classical letter symbols which are very nearly universal in coverage. These comprise the Greek capitals Σ and Π for the sum and product (to which Φ may be added for nonlinear functions - cf. infra), along with the integral sign $\int$ for integration (generally with respect to time). May I now abruptly become quite concrete?

ANALOG SYMBOLISM

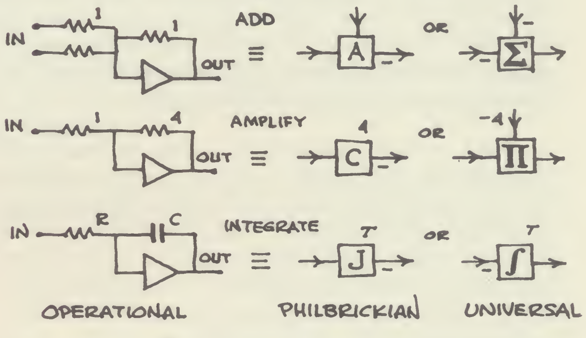


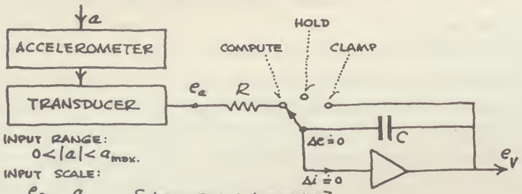
Figure 2

Consider the instrumental problem of measuring the velocity of an object when only its acceleration is known. This may actually be encountered in many practical embodiments, either for rotational motion or linear displacement. We begin with an accelerometer of good accuracy and resolution. You can see that here is an application for an analog integrator.

Now this latter instrument accepts DC voltage as an input, and may be able in typical cases to distinguish as little as 0.001V from zero. Thus if we want fidelity of one part in 10,000 we should require an input voltage range of ± 10V. If the accelerometer will not give this range for the expected range of acceleration, or if its output is of some other character, then our first problem is one of conversion. Suppose we have this problem conquered, and we have a DC output which measures acceleration to the needed accuracy and resolution, and which goes to 10V at the maximum expected acceleration. We have thus successfully "scaled" this portion of the installation. (Fig. 3)

A single conventional analog integrator will now do a superb job of measuring - or computing - the variation in velocity from any assigned starting value. The arrangement may readily be sketched out and explained; I shall not spoil your fun by giving a complete account. This highly useful mechanism is actually quite generally representative of analog computing, and the logical and instrumental steps cover most of those items which are important to a broad understanding.

PRACTICAL USE OF ANALOG INTEGRATOR



INPUT RANGE:
$0 < |a| < a_{max}$.

INPUT SCALE:
$-\frac{e_a}{10} = \frac{a}{a_{max}}$ [INVERT FOR CONVENIENCE]

INTEGRATOR EQUATION:
$e_v = -\int_0^t \frac{e_a}{RC} dt$ [WITH SPECIAL CASES FOR CLAMP & HOLD]

OUTPUT SCALE:
$\frac{e_v}{100} = \frac{v}{v_{max}}$ [BY DEFINITION, $v = \int_0^t a\, dt$]

TO MAKE EQUATIONS AGREE, CHOOSE:
$RC = \frac{v_{max}}{10\, a_{max}}$

Figure 3

One scales the integrator output voltage in terms of velocity by considerations similar to those at the input, chooses proper values for the resistance and capacitance of the network to give an acceptable characteristic time, inserts initial conditions by bringing the output to zero through clamping (or to a known initial value through charging the capacitor or by still different means), stops the integration at a desired point to hold the answer for convenient utilization, etc.

Just as one integrator serves to measure velocity, the addition of a second similar integrator will give displacement, or more precisely its variation from a known or assumed initial value, from the single accelerometer input. It is instructive to think out the steps required for successful operation. It is worth pointing out that this instrumental feat may be successfully reversed, giving velocity and acceleration when displacement is fed in. In this case the initial conditions are greatly simplified. The unit operations, now differentiators, employ reversed positions for the computing resistor and capacitor, plus a couple of minor tricks which you may already know, - or can learn by calling the right number in Boston.

Returning to the cascaded integrators, please note that if the acceleration were that of a single body of known mass (or inertia) then it also measures the net force (or torque) on the body. Now if this body were sprung to a frame, and damped as well, we could synthesize this situation if we wish to, since we have outputs of velocity and position. Imagine the input to be disconnected and replaced by the output of an analog adder, fed itself through amplifiers responding to the velocity and position voltages. The result may be considered either a model of a system with one degree of freedom, or an equation solver. In either case the transition has been made from a measuring instrument to a self-contained, though modest, computer.

HOW INFORMATION IS FURNISHED TO A COMPUTER

When a computing machine is part of a larger system, as for example when it is being applied as a simulator or as a data-reducing instrument, then the inputs entering from the outside are furnishing information in a rather obvious way. Let us consider the less obvious ways information gets in, whether for simulation or for self-contained operation.

Perhaps the most important information we can put into an analog computer is embodied in the internal paths of communication which we permit among its parts, that is to say in the interconnections of inputs and outputs already alluded to. These interconnections determine the kind of equations the computer has to solve, or the sort of structure or phenomenon it must model. The language normally employed for this phase is the block diagram, on which appear the computing units and the conductors carrying their inputs and outputs.

A telephone switchboard is a wonderful general analog for the interconnections of an analog computer. An amusing metaphor or allegory may be traced out, in which the telephones of a town are plugged together for one way conversations, the subscribers being likened to different types of computing units. The fact is that interconnections may be made in many ways. Just now removable patch-boards (see below) are very much in fashion. Next season's style may be different. Please note one inflexible rule on interconnections: namely that the outputs of two or more computing units are never tied together.

With the interconnections properly completed in our computing machine, and the qualitative relations established thus among the variables involved, the next informative operation is to make these relationships quantitative as well. This operation amounts to the setting of Parameters: a set of dimensional factors which determine the condition of the system. Included principally among these factors are the characteristic times of integrators, the gain settings of amplifiers, and the attenuation of potentiometers if used as attenuators. The parameters of nonlinear operations are also involved: the limiting of bounders, the spans of backlash units, and the more complex parameters of functional components.

The procedures of scaling form a large subject which enters in this connection. Without

trying to cover this subject in great detail, we can refer back to the integrated accelerometer already described as a typical scaling situation. More generally, the purpose of scaling is to employ as much of the useful range of each instrumental scale as possible, at the inputs and outputs of the computing units, without running the dangers of saturation on the one hand or probing the limits of resolution on the other. It is evident thatthis leads to optimum accuracy, among other benefits. A knowledge of the primary structure being studied is invaluable for this purpose, and here is one of the reasons for letting the project development man into the room; but of course a clever mathematician does such scaling all the time and will ask the right questions.

Here is a ready rule for scaling. Note first that each variable (voltage) of the computer corresponds to a variable of the "real" problem. Maximum and minimum values of the latter are known, or knowable, and should be tabulated along with ranges of the parameters to be studied. Then in setting up the correspondence between the real variables and the machine variables (voltages), the extreme values of the former should be made to correspond to something short of - but not too far short of - the saturation values of the latter. Simple? Obvious.

An important class of information is called initial conditions. These are either fed in as stimuli from outside the machine, or put into effect by essentially switching operations like those mentioned earlier. Such operations may be carried out by direct manual switching, or through normally directed relays. They may also be done automatically, as is necessary in the case of repetitive computation, in which as soon as a solution is completed another is automatically set up and initiated.

In simulation, the establishment of initial conditions may be necessary in order to put the simulative instrument in a preparatory state analogous to that of the real structure. For many simulators, however, the problem may be either non-existent or much simplified, since their behavior may be so completely under the dominance of the input variables; furthermore, following an initial period of connection, the simulator may operate for arbitrarily long periods with the test conditions delegated to other parts of the larger system.

In connection with setting in parameters, we must point out that some computors now permit the user to set a desired gain on a central decade switch and arrange to "servo-in" this gain through automatic manipulation of a selectable potentiometer. This is an interesting piece of human engineering, but I am afraid my comments on it would be presumptuous, if not unbecoming. It is not available, at least now, from my Company.

I must also speak of so-called problem-boards or patch-panels. These are constructed to carry the interconnections between all computing units, and consequently to facilitate setting up a computation and changing from one problem to another. Ordinarily, however, these boards do not embody the setting of parameters, which may - especially in the instances of nonlinear functions - take as long to establish as the interconnections. The servoing-in of parameters is pointed to in answer to this argument, but if you are going to this intricacy why not "servo-in" the interconnections? This is in fact being done by one of our customers. In this area I think some rapid evolutions - if not revolutions - can safely be predicted. There is a good deal more to be said, although I may already have said too much. In any case I can tell you this: those who value the block-diagram approach to computing, in which the computing units may be arranged in a pattern which is reminiscent of the diagram itself, generally will not go for the patch panel philosophy.

The plug board technique could save time if only one or two solutions were ever run on a given computing set-up. Normally, however, this is a drastic exception. Many sets of parameters are usually tried, in empirical experimentation. By estimates in our own laboratory, the time for purely mechanical setting-up amounts on the average to less than 5% of the total time on a problem, even for the fastest computing machines.

PARTS LIST OF A TYPICAL ELECTRONIC DIFFERENTIAL ANALYZER

This particular part of the present exposition is a heartbreaker for me, since our own credo and policy involves recommending a selection of components which is adapted to the kind of problems which the user wants to handle. But on this show I am submerging proprietary idiosyncracies, and trying not to let the tears show through. On with the show!

A typical modern analog computer will comprise about 30 Operational Amplifiers.

It will have available perhaps 200 assorted precision resistors, of which 30 will be adjustable, and maybe 40 precision capacitors in various sizes. Some 20 individual diodes will be on hand, semiconductor or thermionic.

For taking products of voltages, our typical laboratory will want 6 independent analog Multipliers; for nonlinear voltage functions, 2 Functional Components will have to do.

Naturally, Regulated Power Supplies will be included, to put this equipment in condition to perform.

To initiate solutions, to make various routine checks, etc., a Central Operating Panel will normally be required.

To display solutions, add a meter or two, an oscilloscope, and a direct-writing recorder.

The price of this installation will be between 7 and 27 K$.

An important facility for the typical analog installation is floor space. Of course, if you had a desk-top computer, this would be unnecessary! Again, if the installation is for simulation it may have to be wheeled into some space which was overlooked next to a test rig in the laboratory basement. Now I should quickly state that some wonderful work has been done in cruelly cramped corners.

Seriously, space is a fairly important item, and too much is better than not enough; it serves the human factor, and allows for the inevitable additions. Similarly, air-conditioning generally pays off as a morale benefit, since most computers dissipate more heat than do 50 men.

Speaking of space, one large company (not competing with yours or mine) is putting an analog computer in a large multi-story building. One of the floors is for the Operational Amplifiers. My guess is that the housing, and the space, represents a minor piece in the cost-accounting. Anyhow, we didn't get the job.

STANDARD MACHINES OF THE PRINCIPAL MANUFACTURERS

Whereas about a decade ago there were two manufacturers selling electronic analog computers, there are now more than eight. This does not include makers who have temporarily withdrawn from the market, nor a large group who make only certain analog devices rather than full-dress computing machines.

The best-known names as of this moment, and I sincerely hope I am not omitting anyone, are as follows (alphabetically): Berkeley; Boeing Airplane; Donner Scientific; Electronic Associates; Goodyear Aircraft; Mid-Century Instrumatic; Philbrick Researches; Reeves Instrument; and Short Bros., Ltd., in Northern Ireland.

All of these are ready to sell you a computing machine. I cannot tell you which one to buy. Certainly I must not. However, I have solicited photographs from each of a representative machine. These I can show and make comments on, based on a tabular summary. I shall not try to prove anything in this review, beyond the self-evident fact that we have an abundance of analog apparatus to choose from. (Figs. 4 - 12)

Fig. 4. Ease Model 1032,
Beckman Instruments, Inc.

THE ARGUMENT FOR MODULAR CONSTRUCTION

The accepted reasons for building a computing machine in modular units are similar to those which obtain for any sort of pre-fabricated assemblage. First, an installation should be permitted which is more or less precisely adapted to the needs of the

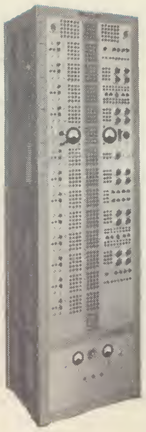


Fig. 5. Model 7000B,
Boeing Airplane Company

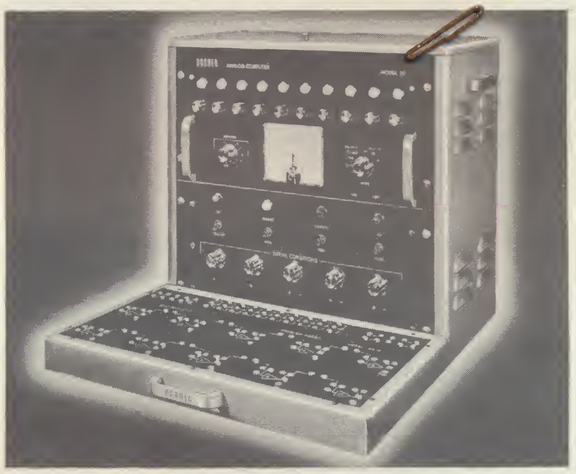


Fig. 6. Model 30,
Donner Scientific Company

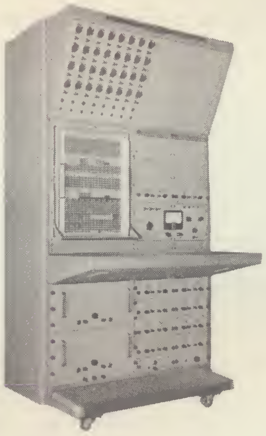


Fig. 7. Pace Series 16-31R,
Electronic Associates, Inc.

application, in terms of the numbers of each kind of subordinate unit. Secondly, the modular form should enable a natural growth of the installation, by the addition of units a few at a time. A further reason is based on a belief not at present as widely held, namely that most of the parts or units of present day analog computers have instrumental applications in their own right, not only in the laboratory, but in testing, in training, and even in production itself. This leads logically to a more articulate kind of modularity, whereby these parts may be applied either singly or in modest com-

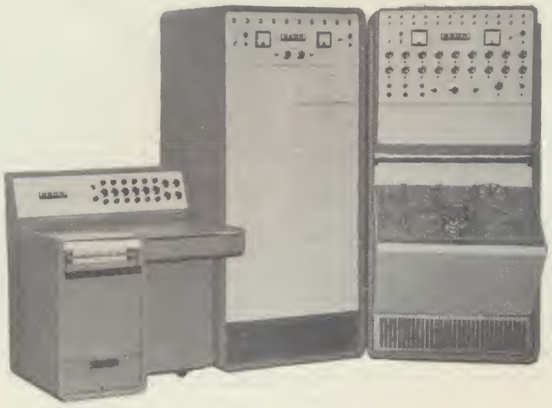


Fig. 8. Geda Models R5, N3, L3,
Goodyear Aircraft Corp.

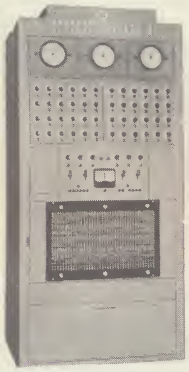


Fig. 9. MC 500,
Mid-Century Instrumatic Corp.

Fig. 10. Siemens Installation (Germany)
Philbrick Researches

binations of units for instrumental purposes remote from and different from the discipline of the normal computing center. This usage has materialized so emphatically that we are now building computing units in a new series of modules which are completely self-sufficient. This auxiliary line of instruments is constructed in a handy and identical size, suitable for desk or relay rack, and is based on the successful development of a relatively small but highly efficient local power supply. Power comes to each module directly from the AC power

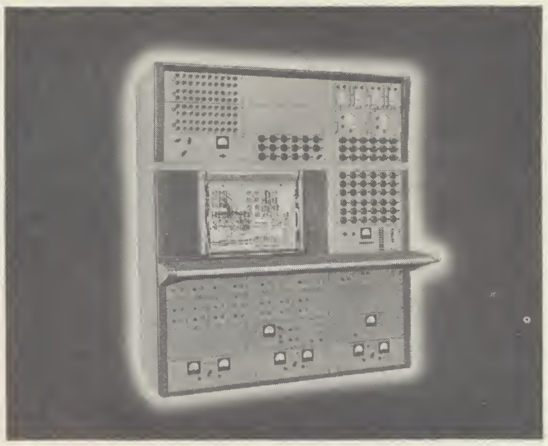


Fig. 11. REAC Series 400,
Reeves Instrument Corp.

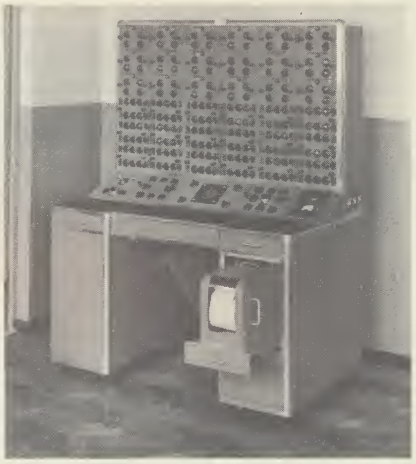


Fig. 12. Type B,
Short Bros. & Harland Ltd.

mains, thus eliminating one of the principal obstructions to complete flexibility, namely the problem of manufacturing and stocking regulated power supplies in many different capacities.

I should perhaps have avoided mentioning this new line, since my mission is to cover the competitive field as broadly as possible. My excuse is simply that we are building several models in this latest modular form for GEL in Schenectady. Included are instrumental items which represent a new and intensive philosophy of analog assemblage and application. For a fuller account I refer you to Mr. S. G. Reque of that laboratory. You may wish to ask him about (a) a novel Operational Amplifier Composite with several basic applications, (b) a revised Nonlinear Functional Component with new features which will soon make practical the embodiment of functions of two or more variables, and (c) an automatically variable Delay and Synthesis modular unit.

Service on separable and self-sufficient components should be greatly facilitated, both for trouble-shooting on location and for factory attention; though instrument makers are not infallible quite yet. All the new models have shipping weights of about 25 lbs. A final virtue is the facility for adding improvements and new models without spoiling the decor and function of the computing ensemble. Change and progress is the order of the day. New techniques and mechanisms cannot

be predicted in detail, but it is entirely predictable that they will keep coming forth.

THE CONTROVERSIAL QUESTION OF SPEED

A great many electronic analog computers operate at a speed established by the fact their integrators have characteristic times of one second, as determined by the very popular resistance of one megohm and the very popular capacitance of one microfarad. This incidentally points up the fact that in a self-contained computer we are free to choose our own time scale, which may be either faster or slower than that of Real Life. If the computing machine is a "simulator", the choice of operating speed is determined in advance by the normal speed of the structure being simulated. I guess it should be noted here that we are using this term in its usual connotation, for broadly speaking a simulator should certainly be permitted to transform the speed along the time dimension. Curiously enough, there are many simulative jobs which tax the upper limit of speed of most types of analog computers, and this situation will become ever more pressing as mechanisms strive to keep pace with our frantic civilization. I was surprised to learn that there are 10-microsecond wave fronts in the responses of nerve-trunks in the human animal. In the computing business, one picks up fascinating lore like that every day.

My own view is that all computing units should be adaptable to operate over as wide a range of speeds as possible. One of the real advantages of electronics is that it permits such pan-celerity. Low speeds are important for simulating slow structures, and high speeds for certain simulation jobs but also for independent computations in which solution time is the bottleneck in finding optimum conditions among many parameters. Now the commonest objection to fast computing is that it introduces limitations on accuracy. The easiest answer to this objection is to say that accuracy need not be limited by speed, but I think the best answer is that a variable-speed machine will permit slowing down to check the accuracy of a solution if it is questioned.

It is well known that we are strong for repetitive computing, so that I shall not dwell on this point, except to point out its unique advantage for solutions which involve random signals such as are now coming into use for the statistical study of controls. To add one more general comment: an exploratory tool should show the prospector where to dig and should do this precisely enough and soon enough to keep him from defacing the whole country-side.

To return for just a moment to accuracy, may I urge you to recognize more than one item in connection with the specification of this quality. As in other instruments which you all use as tools, you should include resolution, fidelity, and precision (or reproducibility) along with absolute accuracy. Look carefully also at the accuracy of non-linear as well as of linear operations, since it is in the former that accuracy is hardest to achieve and where it is most keenly required.

CONCLUSION, IN WHICH I PRESUME TO ADVISE

You will certainly continue to succeed in engineering ventures even if you follow doctrines exactly opposite to mine, but I shall try your patience a little farther with some suggestions which I believe are sound, and which are sincerely distilled from a fairly broad experience with computers under many different conditions.

When and if you make computing facilities available to your engineering staff, set up the lines of origination and action so that your creative engineers may have direct access to the computing machinery itself. In this I am speaking of analog computation; to set up rules for the other types would be presumptuous of me. The best usage is to avoid turning a problem into a numerically specified equation, and then seeking a single numerical solution. For one thing, the machine can better deal with the primary equations themselves, before mathematical consolidation. But more important, for the greatest benefits in typical cases, a whole spectrum of questions will require answers, and one of the vital questions may be "What question should be asked next?" The exploratory procedure is thus generally sequential, or experimental, meaning that there are logical loops which the machine can help to unravel if given a part in the deliberations. The thread of the tale can easily be lost if an organization intervenes between the engineer and his crystal ball.

If you want results, promote a free interchange of ideas with the analog machine. Odd as it may sound, the computer will frequently make suggestions at an engineering level, when an understanding develops. Take it from me, it can even invent. But not by itself.

Above all, be assured that you cannot replace brain power with a computer. No substitute for creative thinking has yet been reported. Nor can the machine convert mediocre technical men into prodigies. The proper application of analog instrumentation is to extend, to augment, and to liberate the developmental and creative engineering brain power which is already available in your departments.

Now may I speak for all analog computer manufacturers, and for all lovers of Analogy as well, who would join me in this if in nothing else, in wishing you good prospecting, and many a rich vein and fabulous Eldorado!

REPRINTED BY

George A. Philbrick Researches, Inc.

127 CLARENDON STREET · BOSTON 16, MASSACHUSETTS

PRINTED IN THE UNITED STATES OF AMERICA

1164N1M Printed in U.S.A.

ANALOG COMPUTER TECHNIQUES APPLIED TO INDUSTRIAL INSTRUMENTATION AND CONTROL*

NOTES ON OPERATIONAL AMPLIFIERS

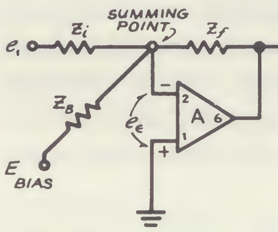
GENERAL

The text for this portion of tonight's presentation is the K2 Applications Manual, a copy of which is incorporated in your notes. These notes are merely supplementary. The Manual itself is the focal point. The operational concept is characterized by amplifiers featuring:

a. Dc amplification and requiring plus and minus power supplies.
b. Phenomenonally large values of voltage gain, such as 10^4 to 10^8 or more.
c. A fairly constant roll-off of the value of gain with increasing frequency. The roll-off is ideally about 6 db/octave until well beyond the frequency at which unit gain occurs.
d. Low or negligible input current, lower than is generally obtained from an open grid.
e. Very low dc drift, generally spoken of as an equivalent spurious voltage in series with the input grid. It is useful to think of it as the minute voltage required at the grid to bring the output voltage back to zero.
f. Input stage, usually of the balanced differential input type, sometimes referred to as a "long-tailed pair".
g. The input voltage required to produce full output is usually insignificant compared to voltages in the circuit as well as compared to the long term drift.

BASIC USES

Think of the operational amplifier as ACTIVATING a network. From the standpoint of applications, it is basically an ACTIVATOR. It may produce attenuation. As an amplifier, it is usually diagrammed as shown in figure 1.



GAIN = $\frac{e_o}{e_i}$ (NOTE: "A" IS A NEGATIVE GAIN TO DENOTE SIGN REVERSAL)

$$\frac{e_o}{e_i} = -\frac{Z_f}{Z_i}\left[\underbrace{\frac{1}{1-\frac{1}{A}\left(1+\frac{Z_f}{Z_i}+\frac{Z_f}{Z_B}\right)}}_{\text{TYPICALLY LESS THAN 0.01\% AT D.C.}}\right] \cong -\frac{Z_f}{Z_i}$$

FIGURE 1. OPERATIONAL AMPLIFIER, General Schematic Diagram

Note that the amplifier is shown as a differential unit. It is implied that the gain is so high that the differential error voltage, e_ϵ, is negligible compared to e_i. The summing point is thus virtually at the same potential as the plus input. In the above case this is ground. The actual observed deviation from this is typically caused by drift alone, because signal requirements are small compared to drift.

Note also that since no current can be drawn by the amplifier, Z_i and Z_f form a voltage divider with the peculiar restriction on e_o that it must be of whatever value is required to keep the junction between those resistors (the summing point) within a millivolt or so of ground (or microvolts when the amplifier is drift-stabilized).

The question usually arises in instrumentation: "How can I prevent loading the source by the input resistor, R_i, which is usually or desirably pretty low in value for high gain?"

Remember that the whole technique revolves around balancing current through very precise impedances. Thus we can amplify positively instead of negatively

*Part II of notes for a talk given before the IRE-PGEC on January 16, 1958, New York City.

(negative is usually spoken of as "inverted operation").

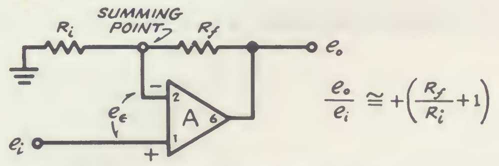


FIGURE 2. AMPLIFIER CONNECTED for POSITIVE AMPLIFICATION

The input voltage, e_i, now sees only the open grid of the positive input which draws negligible current. The output voltage, e_o, must operate the $R_f + R_i$ voltage divider so as to match the summing point to the input voltage.

In typical applications within the current capacity of the amplifier's output, the output voltage, e_o, is very "stiff" indeed. Apparent source impedances can usually be reckoned in microhms or milliohms.

INTERNAL DESIGN CONSIDERATIONS

The first stage of an amplifier is almost always of necessity of the differential input type. For critical or low drift level applications, it is preferable to have at least one balanced stage (differential-input --- differential-output) ahead of the differential input --- single-ended-output stage. (This is done in the Philbrick USA-3's main amplifier portion, and thus provides a very effective guard against any power supply fluctuations causing drift of the output.)

In order to reduce the dc drift to a value several magnitudes less than that of which vacuum tubes are capable, it is common practice to drive the positive input with a chopper-type "stabilizing" amplifier. These "preamplifiers" are inverting (hence work into the positive sign input terminal) and have typically a dc gain of from 500 to 50,000. The stabilizing amplifier looks at the summing point error, and its output drives the positive input of the operational amplifier as necessary to wipe out this error voltage. This effectively multiplies the dc gains of the two amplifiers.

Chopper type amplifiers are really modulated carrier systems; their drifts can, therefore, be made very small. Ten microvolts dc is not unusual, though rarely necessary in well designed computing systems. About 50 μv could be thought of as "design center", and 100 μv as a safe design figure, even though such an amplifier itself may be responsible for only 10 μv and the remaining 90 μv forced upon it by associated circuitry. In a refined operational amplifier for use in very low drift integrator circuits, the connections are made so that no grid current is taken from the input. This is made possible by an ingenious though fairly conventional "zero grid current circuit". This circuit is explained in detail under "Notes on Integrators", to be found further along in the notes.

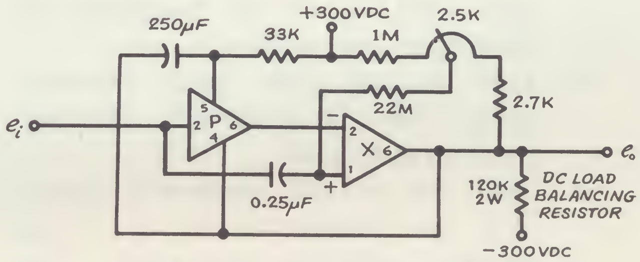


FIGURE 3. STABILIZED FOLLOWER CIRCUIT

It is possible to stabilize a follower, although this is not mentioned in the K-2 Manual. To accomplish this stabilization, however, the operational amplifier must be made to "drive" the stabilizer ground terminal including all of its plate power requirements. Actually this is well within the capacity of most amplifiers when the total load it sees is balanced (so that it furnishes zero dc quiescent current). Additional circuitry is necessary to prevent errors when the signal frequency is almost exactly equal to the power frequency.

Where very high accuracy is sought, it is important to insure that the output is not commanded to swing at a faster rate of change of voltage than the amplifier can comfortably handle. One volt per microsecond is an average figure, some amplifiers being slower (K2-XA, P2, etc.) and some much faster (USA-4JX, etc.). Note that this is not a function of the frequency at which unity gain occurs but rather a function of internal quiescent currents.

SOME THOUGHTS ON NOISE AND PICKUP IN OPERATIONAL AMPLIFIER CIRCUITS

GENERAL

Noise apparent in operational amplifiers falls into two separate and distinct classes:

Class A: Noise in output due to an apparent NOISE VOLTAGE in series with the input grid.

Class B: Noise in output due to an apparent NOISE CURRENT fed into the summing point.

To define the noise performance of an operational amplifier in a general way requires a description of both classes; one is not enough, as will be seen.

CLASS A NOISE

To illustrate Class A noise voltage, consider noise which is actually introduced in, say, the second stage of the amplifier. As users of the amplifier, we might ground the summing point (See figure 4.) and observe the result as a noise voltage in the output. One can then say (and quite properly so) that this is exactly the equivalent of a noise voltage inserted in series between the summing point (now grounded) and the input grid of the amplifier. It can be cancelled out by an equal and opposite voltage between the summing point and ground. But in no way does it resemble a current at the input grid or summing point. It is a true equivalent voltage.

CLASS B NOISE

The bulk of circuitry in which operational amplifiers are used shows surprisingly low noise, but what there is generally is of Class B. To illustrate Class B noise current, let us refer again to figure 4.

Consider the unbalanced capacitance between the input grid and the heaters (typically grounded on one side) as represented by e'_N and C'_N. Noise CURRENT then is fed into the summing point. If one were to physically ground the summing point, then none of this Class B noise would be observed in the output. But in normal operation, the summing point is perhaps within microvolts of ground potential, the difference being the error voltage needed to actuate the amplifier. Any current entering the summing point leaves by the sole path through R_f in figure 4. Now one observes a noise voltage in the output which is the product of the feedback impedance and the Class B noise current.

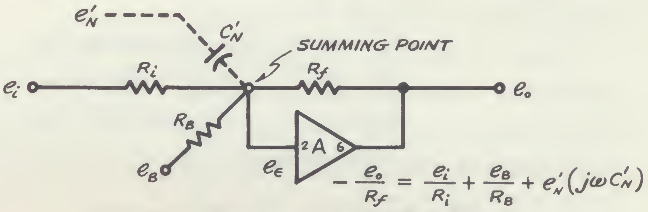


FIGURE 4. TYPICAL AMPLIFIER CIRCUIT

Let us now apply these concepts to a typical circuit, in a more quantitative manner. Figure 4 represents a typical amplifier, including a bias voltage, e_B, and the inevitable small capacitance, C'_N, coupling the summing point to some arbitrary voltage, e'_N, (often 60 cycle heater circuit voltage). Although some circuit components are shown as resistances, exactly the same calculations would apply if any or all components were expressed as complex impedances.

In the circuit, of figure 4,

$$-\frac{e_o}{R_f} = \frac{e_i}{R_i} + \frac{e_B}{R_B} + e'_N \left(j\omega C'_N\right)$$

But the attenuation from the amplifier output to its input, i.e., the amount fed back, β, is determined by the voltage divider effect shown in figure 5.

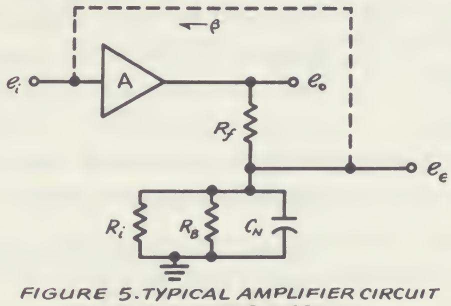


FIGURE 5. TYPICAL AMPLIFIER CIRCUIT VOLTAGE DIVIDER EFFECT

To correct a noise disturbance occurring somewhere in the amplifier, the output voltage, e_o, must be large enough to make e_ϵ equal and opposite to the disturbance. The "noise gain" is then $1/\beta$.

The only safe way to view an operational amplifier circuit is to consider it a current balancing scheme in which the algebraic sum of currents into the summing point must equal the current through R_f. Therefore, let us consider:

A) If noise were an induced voltage in R_f, but none in R_i or A:

The noise e_N picked up in R_f would appear unamplified in e_o and should be negligible.

B) If the noise were an induced voltage in R_i, but no other place:

No noise voltage components of significance in e_ϵ.

The noise would be amplified, but only by the ratio R_f/R_i.

C) If the noise were an induced voltage in R_B, the action would be like that in B.

D) If the noise were a current fed into the summing point by capacitive coupling (See figure 4.) it would appear in e_o amplified since this noise current must be balanced out by an equal current through R_f:

$$\text{noise in } e_o = \left(j\omega C'_N R_f\right) \times e'_N$$

E) If the noise were picked up anywhere within the amplifier, it could be treated as being effectively a noise voltage in series with the grid of the input stage.

Noise appears undiluted in e_ϵ.

Such noise would appear in e_o amplified:

$$\text{noise in } e_o = e_N \left[1 + \frac{R_f}{Z_x} \right]$$

where Z_x = paralleled value of all impedances effectively between summing point and ground, i.e., R_i, R_B, C'_N, etc.

CAPACITIVE PICKUP

Consider an example of capacitive pickup:

Let: ω = 377 radians per second (for 60 cps operation)

e'_N peak to peak = 6.3 vac $\times$ 2.8

R_f = 1 Megohm

Then noise e_o per micromicrofarad coupling to the heater supply will be:

$$e_o = 377 \times 10^{-12} \times 6.3 \times 2.8 \times 10^6 \times 10^3 = 6.65 \text{ mv}/\mu\mu\text{f}$$

This noise is not too bad.

Now let: ω = 2500 radians per second (for 400 cps operation)

e'_N peak to peak = 115 vac $\times$ 2.8

R_f = 10 Megohm

Then noise e_o per micromicrofarad coupling to the power line is:

$$e_o = 2500 \times 10^{-12} \times 115 \times 2.8 \times 10^7 = 8.05 \text{ v}/\mu\mu\text{f}$$

Such noise is usually objectionable, though not always.

This example shows the severe effect of the slightest capacitive unbalance in the tube heaters and associated heater circuitry, or of inadequate shielding of the summing point when high impedance feedback circuits are used.

INDUCTIVE PICKUP

Inductive coupling results in the voltage being effectively induced in series with the leads or components. Calculation of this voltage is elementary if the field, i.e., the flux, can be defined. The standard

transformer equation can be applied, namely:

$$e_N \text{ (peak to peak)}$$
$$= e_N \text{ (rms)} \times 2.8$$
$$= \frac{4.44 NfBA}{10^8} \times 2.8$$

Where N = turns (typically one)
f = frequency of noise harmonic being considered.
B = peak flux density (typically Gauss)
A = area enclosed (in square centimeters)

If a 3 foot (100 centimeter) length of lead runs 4 inches (10 cm) on the average away from its return lead, it constitutes one turn and an area of $A = 10^3$ sq cm. Then a field of 10 gauss rms (which is 14 gauss peak) at 60 cycles will induce:

$$e_\phi = \frac{4.44 \times 1 \times 60 \times 14 \times 10^3}{10^8} \times 2.8$$
$$= 37 \text{ mv peak to peak/Gauss rms}$$

How objectionable this noise is depends upon the application. If the lead happened to be the one running from the summing point to the input of the operational amplifier operating at a gain of 10, such a value of noise, 37 mv becomes 370 mv at the output, and is about 300 times larger than the inherent noise level possible to achieve using ac heaters.

There are obvious fixes for inductive pickup, but all too often they are applied haphazardly, or without enough understanding. The three standard quick-fixes are:

1. Twisting lead with its return, hoping to have induced in each twist a voltage equal and opposite to the voltage induced in the previous twist.
2. Running the lead inside a hollow return path (co-axial philosophy), hoping that the lead and its return will have equal and opposite voltages (referred to the circuit) induced in them.
3. In grounded circuits, shielding the lead with heavy copper braid and grounding both the originating and the terminating ends of the shield through low impedance connections. This forms a so-called "shorted turn", which, if of low enough resistance will force the flux to go elsewhere.

"Humbucking" has been successfully used, but it is certainly the hard way to improve the situation. The easier path when all else fails -- the final resort -- is dc on the heaters and complete shielding from noise sources.

NOISE RESULTING FROM CHOPPER CORRECTION

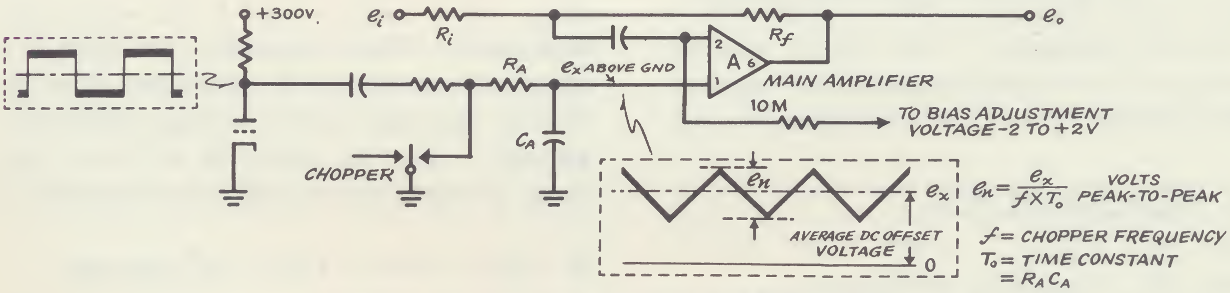


FIGURE 6. APPLICATION of CHOPPER STABILIZATION

Offset in the main amplifier is washed out by the action of the stabilizing preamplifier. Its output moves away from zero by an amount nearly equal to the offset of the main amplifier. However, this comes from the plate of the preamp in the form of square pulses which must be averaged into a pretty smooth dc bias by $R_A C_A$, to correct out the main amplifier's offset.

Averaging is never perfect, and the lack of perfection here is simply a triangular wave of "noise" at the chopper frequency. This is small; in most applications it can be neglected, but not until it has been established that it is, <u>in fact</u>, negligible. With no bias control, it can easily be imagined that from one tube sample to another, offset variations may be as large as 0.3v independently of the preamp gain.

Let: Chopper frequency = 60 cps
R_A = 22 Megohm
$C_A = 1\mu f$

Then: $e_n = \dfrac{0.3}{60 \times 22 \times 1} = 225\ \mu v$ peak to peak

From this it can be clearly seen why an adjustable bias on the main amplifier is desirable for low noise circuits.

INTEGRATOR CIRCUIT NOTES

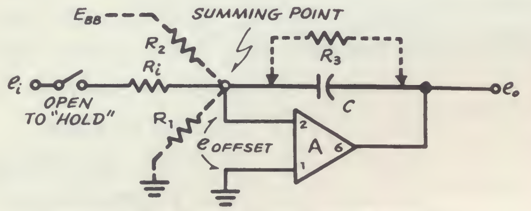


FIGURE 7. TYPICAL INTEGRATOR CIRCUIT

An integrator can be extremely precise. Consider a 0.1% loss of voltage in 1000 secs when used to "hold" a voltage; i.e., to "remember" it.

Let: T = 1000 sec
Allowable loss in time T = 0.1%
$C = 0.1\mu f$
e_o = 50 volts

Then: Rate of loss $= \dfrac{dv}{dt} = 50 \times 0.1\%/1000$ sec
$= 50\ \mu v$/sec ave.
i = loss current for the assumed holding ability
$= C\dfrac{dv}{dt} = 0.1 \times 10^{-6} \times 50\ \mu v/\text{sec}$
$= 5\ \mu\mu a$

Now consider the various errors:

a. <u>Error caused by R_1</u>

Let: R_1 = 2 Meg
Then: $iR_1 = 5\mu\mu a \times 2 \times 10^6\ \text{ohm} = e_{offset}$
Permissible $e_{offset} = 10\ \mu v$
Let: Amplifier gain $= 25 \times 10^6$
$e_{error\ signal} = 50/(25 \times 10^6)$
$= 2\ \mu v$

Permissible drift $= 10\ \mu v - 2\ \mu v$
$= 8\ \mu v$ during the 1000 second problem time.

b. <u>Error caused by R_2</u>

Great care must be exercised to shield the summing point and its leads from leakage paths to other voltages. Insulation at best is not likely to be adequate.

If E_{BB} is, for example, the plate supply, typically 300v:

$$\frac{300v}{5\mu\mu a} = \frac{300 \times 10^{12}}{5} = 60 \text{ million megohms}$$

This is larger by a factor of one million than good design practice can justify. By shielding, all leakage can be kept to ground and represented by R_1.

c. <u>Error caused by R_3</u>

There is always some leakage in a capacitor, but it may be kept to an extremely low value. If the capacitor is built in a shielded can which is grounded, then R_3 will be due only to the leakage of the capacitor. For an error of 0.1% in 1000 secs it must have a self time constant of

$T = R_3C = 1000 \times 1000 = 10^6$ seconds

If $C = 0.1\mu f$, then $R_3 = \dfrac{10^6}{0.1 \times 10^{-6}}$
$= 10^{13}$ ohms
or 10 million megohms

This is not an unreasonable value for a

computor type polystyrene or mylar capacitor of this size.

d. <u>Error due to grid current</u>

The foregoing assumes a grid current small compared to $5\mu\mu a$. Since conventional vacuum tubes exhibit grid currents of ten to a hundred thousand times larger, special circuits must be resorted to within the amplifier.

The main amplifier is made a capacitor coupled ac amplifier. The dc to low frequency signals are handled by carrier modulation techniques requiring again a dc blocking capacitor and a megohm resistor to ground half the time. (See figure 8.)

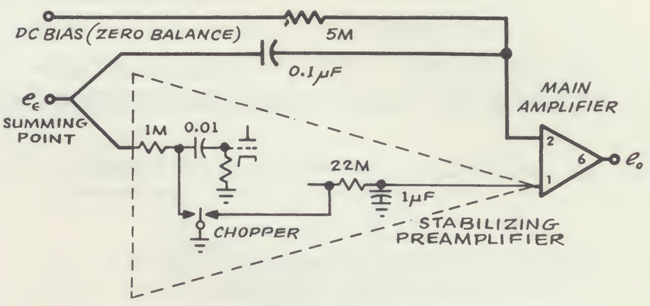


FIGURE 8. ZERO GRID CURRENT ARRANGEMENT

e. <u>Noise</u>

The chief objection to noise is that the 60 cycle or harmonic components confuse the preamplifier chopper circuitry. Their presence produces offset, but it can be compensated for by dc bias provided it does not vary.

DIFFERENTIATOR CIRCUIT NOTES

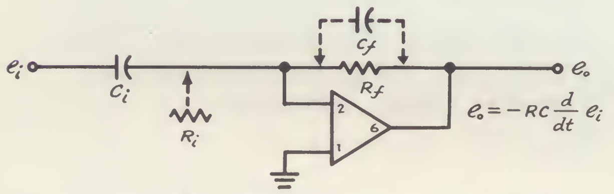


FIGURE 9. TYPICAL DIFFERENTIATOR CIRCUIT

Differentiators are inherently troublesome compared to integrators. They tend to be noisy and unstable, since the noise in e_i, especially higher frequency noise, is delivered to the grid through the relatively low impedance path C_i, and the feedback is delayed by R_fC_i.

The solution recommended is to make it no better a differentiator than is actually required.

If a 1% differentiator is desired, take half the error by putting a resistor in series with C_i and the other half by a capacitor in shunt with R_f.

To do this quickly, consider the highest frequency which must actually be passed with a 1% error. Choose R_iC_i and R_fC_f such that their "natural" frequencies will be 10 times the frequency at which 1% accuracy is wanted:

$$\frac{1}{R_fC_f} = \frac{1}{R_iC_i} = 10(2\pi \times \text{freq for } 1\% \text{ error.})$$

At a frequency which is lower by a factor of the square root of ten, the error will be only 1/10%, etc., since the error decreases with frequency squared, to a close approximation.

To be more specific in details, the "error factor" by which the true theoretical derivative would have to be modified would be simply:

$$\text{error factor} = \frac{1}{(1 + T_p)^2}$$

where $T = R_iC_i = R_fC_f$

0.01% CLASS WITH INTERNAL RESISTANCE OF LESS THAN A MILLIOHM

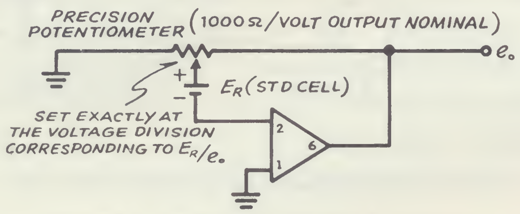


FIGURE 10. PRECISION VOLTAGE SUPPLY

THUS for a 100K precision divider and 100 volts output, set the divider percentage at the exact standard cell voltage, eg., 1.01830v.

Since drifts of 5 to 10 millivolts would not be satisfactory unless large reference voltages are used (45 v batteries or mercury "B" batteries), a stabilized amplifier is indicated, such as a K2-P/K2-W combination, or a USA-3. After proper zeroing, the offset will be less than 100 μv.

Therefore, the expected error will be less than 100μv out of 1.0830 volts (Weston standard cell), which is. less than 1/100%, plus the deviation from perfection of the divider.

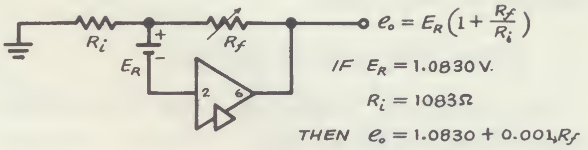


FIGURE 11. STABILIZED PRECISION VOLTAGE SUPPLY

George A. Philbrick Researches, Inc.
127 Clarendon Street, Boston, Massachusetts 02116

964A10 Printed in U.S.A.